Teachers' Choice of Tasks: A Window Into Beliefs About the Role of Problem Solving in Learning Mathematics

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This paper reports part of a larger study into primary school teachers' problem-solving beliefs and practices in NSW. In particular, teachers' selection of problem-solving tasks and the reasons for their choices were investigated. Teachers reported choosing exercises and application problems more frequently than open-ended or unfamiliar problems. Teachers preferred to use exercises for practice of basic skills and procedures, and application problems as indicators of the relevance of mathematics. Open-ended and unfamiliar problems were considered to be appropriate for more able students or for students in higher grades. Reasons given for task choices revealed particular beliefs about how students learn mathematics.

Given the amount of policy advice and resource development, there are concerns about the limited implementation of problem-solving approaches in mathematics classrooms (e.g. Pegg, 1997). To support teachers in implementing problem solving, curriculum documents have been advocating the use of problem-solving approaches since the late Eighties (Stacey & Groves, 1989). Some syllabuses include examples of problems and investigations to assist teachers in their planning of lessons and programs of work (e.g. NSWDE, 1989); however, there is evidence to suggest that providing an innovative curriculum that advocates a focus on problem solving and investigative approaches does not necessarily change teachers' practice (Norton, McRobbie, & Cooper, 2002).

It has been argued that beliefs impact on teachers' practice (Thompson, 1992) and that particular beliefs about problem solving have created a lack of acceptance of problem-solving teaching approaches (Grouws, Good, & Dougherty, 1990). In this study, it is anticipated that teachers' responses to survey questions about the role of problem solving in learning mathematics, about the types of mathematics problems that are appropriate for classroom use, and for what purposes particular questions could be used, might provide an insight into teachers' beliefs. Identifying teachers' beliefs about the usefulness of certain types of mathematics tasks or questions, particularly those recommended in the literature and curriculum documents, may help to determine why some teachers have not responded to the advice. Those beliefs that appear to impact on teachers' choice of tasks could then be discussed and challenged in preservice and inservice education programs.

Background

It has been suggested that teachers' knowledge and beliefs about the discipline of mathematics, teaching mathematics, and learning mathematics impact on classroom practice. Studies conducted by Fennema, Carpenter and Peterson (1989) indicated that when teachers have a sound knowledge of the content to be taught, they use a wider variety of problems, allow more discussion, and respond more readily to student questions. Teachers who were not confident with the subject matter relied on the text, directed the instruction, and allowed less discussion. This suggests that teachers' knowledge and confidence may be factors that influence adoption of problem-solving approaches.

Stigler and Hiebert (1999) found that differences in the structure of lessons and the types of tasks used by teachers in American, Japanese and German classrooms were related to their

beliefs about the teaching and learning of mathematics. A comparison between teachers from the first two countries revealed that American teachers focused on developing skills with most time spent on practising routine procedures, whereas Japanese teachers focused on developing conceptual understanding with as much time spent on solving challenging problems as practising skills. The problems required students to apply concepts in new situations that were not immediately obvious, or to invent something new. Japanese teachers valued a variety of solution methods to problems and also encouraged students to pose problems for others to solve. Groves and Doig (2002) also found differences in lesson structure and choice of tasks between a lesson presented by a Victorian teacher and that of a Japanese teacher. Stigler and Hiebert (1999) argued that the differences between American and Japanese approaches to teaching mathematics could be explained by differences in their beliefs. One indicator of this was found when 61% of American teachers reported that the main thing they wanted their students to learn from lessons was developing skills and performing procedures while 73% of Japanese teachers wanted students to "see new relationships between mathematical ideas" (p. 90).

Choice of task is an important factor in teachers preparing learning experiences for students. It is proposed that choice of task is one indication of teachers' beliefs about mathematics and how students learn mathematics. In an earlier study that explored teachers' problem-solving beliefs and practices, the author (Anderson, 1996) asked teachers to record two favourite problems and to describe why they like using them. Of the 39 surveyed teachers, only 23 provided at least one favourite problem suggesting the others may not use problems in their regular teaching. For the problems that were provided, several efforts were made to classify them into groups to aid analysis of data. This was quite difficult as many were very general and it was often unclear what the purpose of using the problem might have been. Teachers' reasons for choosing particular problems were grouped into affective variables, teaching factors, learning factors, and problem characteristics, providing some indication of the issues that might impact on teachers' choices. As teachers appear to hold different views about the meaning of the term "problem", it was determined that, definitions would be provided for the types of tasks or student questions teachers may choose to use in typical mathematics lessons.

A list of commonly used mathematics question types was established with reference to teachers' responses to the earlier survey instrument as well as reference to the literature. Hembree (1992) examined a large number of studies about problems and noted that the questions ranged from traditional word problems to non-standard problems. Schoenfeld (1992) suggested that "*real* problem solving" involved working on problems that were unfamiliar, and not necessarily related to the particular mathematics topic currently being studied. Several attempts have been made to classify problems into groups according to recommended purposes (e.g. Clarke & McDonough, 1989). Some of the early classifications included questions that were exercises but more recent listings focus on questions that require higher-order thinking and that may present a blockage for students.

For the purpose of this study, primary school mathematics questions were classified as exercises, application problems, open-ended problems, or unfamiliar problems. Exercises required the application of a known fact or mathematical procedure and would typically be used for practising skills. Application provided examples of use of the topic being studied. Open-ended problems have several solutions with potentially different ways of finding and recording solutions. Unfamiliar problems are not open-ended and do not necessarily relate to

In the *Mathematics K–6* (NSWDE, 1989) syllabus being used in NSW primary schools, teachers are provided with advice about what constitutes a problem, sources of problems; approaches to teaching problem solving including "teaching for", "teaching about" and "teaching through" problem solving; and a variety of teaching strategies. The mathematical content is described in three strands-Space, Measurement and Number-with substrands focusing on particular topics. Teaching and Learning Units for each substrand are organised under objectives with recommended sample teaching activities and problems. In 1998, a supplementary document (BOSNSW, 1998) was released that included a fourth strand, Working Mathematically, with six processes–questioning, solving problems, communicating, verifying, reflecting, and using technology.

Methodology

The data reported here were collected during a larger investigation into primary school teachers' problem-solving beliefs and practices in NSW classrooms (Anderson, 2000). Data were collected from 162 primary school teachers in NSW to a questionnaire based on similar instruments developed elsewhere (e.g. Peterson, Fennema, & Carpenter, 1987). This paper focuses on the data collected from teachers about their use of particular student question types and the reasons for such choices. The questionnaire included a combination of closed and open questions that referred to the student question types described in the previous section. To assist teachers, a set of student question types with examples was presented at the beginning of the questionnaire as Background Information (see Figure 1).

Background Information:

For the purposes of this survey, the following definitions are given to assist understanding of the terms that are used.

After teaching 2 digit addition students could be asked to answer the following:

Type of Student Question:	**Example:**
Exercise	37
(we are not calling this a problem)	+ 34
Application problem	If there are 34 oranges in one box and 37 in another box, how many oranges are there altogether?
Unfamiliar problem	The sum of my mother's age and my father's age is 71. My father is 3 years older than my mother. How old is my mother and how old is my father?
Open-ended problem	□ □ + □ □ = 7 1 — What might the missing numbers be?

Figure 1. Background information about student question types.

Several items on the questionnaire were designed to explore teachers' use of particular student question types and the reasons for those choices. The first question asked teachers "How often do you use each of these different types of questions in your teaching?"

followed by a second question, “From your responses to the above question, briefly describe why you prefer to use those particular types of problems”.

Results and Discussion

Teachers’ responses to the first question relating to frequency of use of particular question types are presented in Table 1. The data indicate that about 70% of teachers often use exercises and application problems in their teaching of mathematics while 20% often use open-ended problems and 11% often use unfamiliar problems. A total of 80% of the surveyed teachers rarely, or sometimes use open-ended problems and 89% rarely or sometimes use unfamiliar problems. These data suggest that teachers have a preference for using standard, textbook type questions on a regular basis in their classrooms.

Table 1
Frequency of Use of Student Question Types (%), N=162

Types of Questions	Rarely	Sometimes	Often
Exercises	5	27	68
Open-ended Problems	22	58	20
Application Problems	4	26	70
Unfamiliar Problems	37	52	11

It was anticipated that the data from the second question would provide information about several important aspects of this investigation. On the one hand, it was of interest to ascertain why exercises and application problems are considered by teachers to be the most appropriate student question types for regular classroom use. However, on the other hand, it was of interest to discover why some teachers have responded to the advice since 20% report that they often use open-ended problems and 11% often use unfamiliar problems. In addition, this investigation aimed to explore what determines teachers’ choices, and what aspects of teachers’ lives impact on their decision making in relation to problem solving.

Analyses of data from this open-ended question were performed in two ways. First, teachers’ comments were grouped according to their relevance to each of the four student question types and then categorised into themes. Second, responses to the first 25 questionnaires received were read and comments categorised according to the main issues discussed. As a result of this process, several categories emerged and these were then used to organise the remainder of the data.

From the first level of analysis, seventy-three respondents (45%) made comments about their use of exercises in mathematics lessons. These comments were broadly grouped into nine categories. The first included comments about the role of exercises in providing practice of basic skills or as a building block to do other problems. The next category included comments about the ability of the students that incorporated remarks about poor language skills. The remaining categories included comments about affective factors such as feelings of success or anxiety, the age or learning stage of the students, confidence of the teacher, curriculum requirements, assessment strategies, parents’ expectations, and accessibility of exercises.

Several of the reasons related to the students in the teacher’s class with many comments beginning with a reference to their current class or mathematics group. For many of the

learning. An interesting comment was made by an experienced teacher of an upper primary grade, that encapsulates several of the identified categories. She reported that she often uses exercises, sometimes uses open-ended problems and application problems, and rarely uses unfamiliar problems. Her response was

> It's safer - children feel more comfortable if they're not made to think. I realise this is cynical - but for many children with low IQs and poor/non existent English language skills, the concept of problem solving is alien. Also it takes up too much time and there is great pressure to "get through" the curriculum. So whilst in theory I acknowledge the potential of problem solving, in reality with some clientele it's too hard.

The data highlight many issues that relate to the selection and frequency of use of exercises in mathematics lessons. Comments indicate that teachers believe that exercises provide practice in basic skills and procedures, particularly for lower ability students and for children in lower grades. Also, exercises are considered to be a part of the curriculum, can be used to assess understanding, and enable children to experience success. Teachers feel confident using exercises in their teaching and are able to readily access sets of exercises for student use thus saving valuable preparation time. These issues are important to teachers and may explain apparent lack of adoption of problem solving in classrooms.

Thirty teachers (19%) made comments about their use of application problems. These comments were broadly categorised into applications or real-life contexts, language issues, age of students, thinking skills, affective factors, teaching factors, and ability of the students. The main reason given for using application problems was the use of real-life contexts to show how mathematics can relate to students' experiences. An issue that was mentioned by quite a few teachers was the language difficulty that many students experienced when trying to interpret such problems. Interestingly, two teachers mentioned that they used these problems as a means of helping students come to terms with language use in mathematics. In common with the comments about the use of exercises, teachers also mentioned the age and ability of students as well as affective factors.

Comments about the use of open-ended questions in mathematics lessons yielded a set of categories with similar considerations to some of the above; however, there were interesting differences. Twenty-three teachers (14%) provided comments which were grouped into the categories of the ability of students, factors relating to learning or thinking, affective factors, the age of students, the accessibility of questions, variety of appropriate questions, confidence of the teacher, and the need to prepare students for mathematics competitions. Table 2 indicates the number of teachers who commented on each category and provides an example of each.

Teachers' comments indicated that open-ended problems were considered to be challenging and were therefore suitable for more able students. They were also considered to be more appropriate to extend the knowledge and experiences of students in higher grades, for use in cooperative groups, and to develop higher levels of thinking. Interestingly, three teachers commented that open-ended questions were suitable for all students since they can work at their own ability level to achieve at least some of the outcomes of the question. It was also reported that these questions were used for variety in lessons although they were rarely available in textbooks and teacher resource books.

Thirty teachers (19%) made comments that related to the use of unfamiliar problems. These comments were grouped into several categories including those that related to the ability of the students, learning and thinking factors, affective factors, the age of the students, factors relating to the language involved in the question, the confidence of the teacher, and the

accessibility of such questions. Reasons given for infrequent use of unfamiliar problems included the notion that these problems were challenging and therefore only suitable for the most able, or gifted and talented students. Also, these questions were usually linguistically difficult and required perseverance that can often lead to frustration. In addition to these factors, three teachers mentioned their lack of experience or confidence in using such problems.

Table 2

Categories of Comments Relating to the Use of Open-Ended Problems With the Number of Teachers' Who Made Comments About Each Category and an Example, N= 23

Category	Number of Teachers	Example of Comments
ability of students	10	Open-ended used more so for GATS (gifted and talented) children.
learning/thinking factors	6	Open-ended problems allow children to bring their own knowledge and strategies to the task as well as respond at their own level.
affective factors	4	They experience frustration with open-ended problems. I try to ensure they have more success than frustration in maths lessons.
age of students	3	Tend to use more open-ended problems with older students.
accessibility of questions	2	These types are not often in my math text that I use with my class - so occasionally I make some up.
variety of questions	1	I like to vary what I use.
teacher confidence	1	Feeling a little bit out of depth with some open-ended problem types even though I think that they have merit in the overall maths program.
preparation for competitions	1	We enter all competitions and do very well. The open type are common in competitions.

For those teachers who reported using unfamiliar problems on a frequent basis, reasons included the desire to develop students' higher level and lateral thinking skills, to challenge and motivate more able students, and the need to teach new problem-solving strategies. These teachers were usually teachers of upper primary grades or those who reported that they were responsible for more able students in the grade.

The second level of analysis of the data involved the categorisation of responses into common themes. This clarified earlier findings and further highlighted the key issues that impact on teachers' decision making. The five main areas of students, school, planning, teachers and question characteristics were identified. Comments relating to each of the five broad categories were subdivided into several focus areas. Students were discussed in relation to ability, learning, age and affective factors. School comments included those about streaming, or ability grouping, and other aspects of schools including assessment procedures, mathematics competition preparation, and parents' expectations. References to the syllabus, textbooks or other sources of problems, and the planning of lessons were grouped under the

category of planning. Comments about the teacher usually related to experience or confidence. Finally, many comments related to question characteristics including language and the purpose of each of the mathematics question types. This classification of teachers' responses is represented in Figure 2.

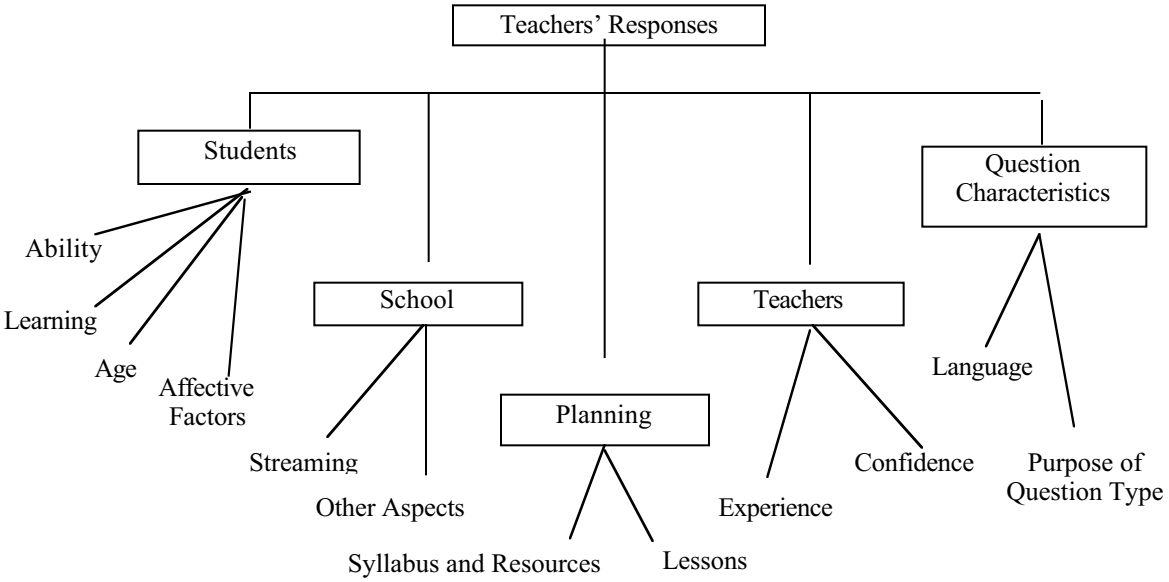

Figure 2. Classification of teachers' comments about reasons for their use of particular student question types.

Given the focus of this paper, it is appropriate to briefly summarise the comments teachers made that were placed in the category "purpose of question type". Twenty-two teachers (14%) commented specifically on the purpose of particular questions in mathematics. It was suggested by teachers that exercises could be used to practise skills, application problems provided opportunities for students to apply their knowledge in real-life contexts, open-ended and unfamiliar problems were usually used to challenge and extend more able students. Several teachers indicated that all types have their place in the teaching and learning of mathematics.

Conclusion and Implications

Teachers choose to use a range of mathematics questions for particular purposes. From this investigation, it appears that teachers' choice of tasks in mathematics lessons is determined by several factors. The experience and confidence of the teacher is clearly a determining factor as well as school-based factors such as ability grouping practices, assessment procedures, parent expectations, and resource availability. Of particular interest were the comments made by many teachers about the purpose of different question types that represented particular beliefs about the role of problem solving for students of different ages and abilities. A small number of the surveyed teachers indicated that *all* students could learn by doing open-ended and unfamiliar problems on a regular basis and appeared to have responded to the advice. Teachers' responses to the survey items provided a window into beliefs about the role of problem solving in learning mathematics.

If the development of problem solving is a goal for all students, this may not be achieved while teachers hold particular beliefs about the way students learn mathematics. Providing advice in curriculum documents does not appear to be sufficient for teachers to embrace such approaches. Opportunities need to be provided for teachers to develop their knowledge and understanding of problem-solving approaches, to reflect on their practice, and to explore a

variety of mathematical tasks to determine how these could be used to engage all students in meaningful ways regardless of perceived ability.

This may not be as simple as it sounds. As Stigler and Hiebert (1999, p. 11) suggest

> Teaching is a cultural activity. We learn how to teach indirectly, through years of participation in classroom life, and we are largely unaware of some of the most widespread attributes of teaching in our own culture.

Perhaps it is time to begin to reconstruct the culture of mathematics teaching in primary classrooms in NSW. To achieve this, research efforts will need to provide teachers with a clearer picture of their role in problem-solving classrooms and to describe the implications for classroom practice (Lester, 1994).

References

Anderson, J. (1996). Some teachers' beliefs and perceptions of problem solving. In P. Clarkson (Ed.), *Technology in mathematics education* (Proceedings of the 19th Annual Conference of the Mathematics Education Research Group of Australasia, Melbourne, pp. 30-37). Melbourne: MERGA.

Anderson, J. (2000). *An investigation of primary school teachers' problem-solving beliefs and practices in mathematics classrooms.* Unpublished doctoral thesis, Australian Catholic University.

Board of Studies NSW (BOSNSW) (1998). *Mathematics K-6 Outcomes and indicators*. Sydney: BOSNSW.

Clarke, D. J., & McDonough, A. (1989). The problems of the problem solving classroom. *The Australian Mathematics Teacher*, *45*(3), 20-24.

Fennema, E., Carpenter, T. P., & Peterson, P. (1989). Teachers' decision making and cognitively guided instruction: a new paradigm for curriculum development. In N. F. Ellerton & M. A. Clements (Eds), *School mathematics: The challenge to change* (pp. 174-187). Geelong: Deakin University Press.

Grouws, D. A., Good, T. A., Dougherty, B. J. (1990). Teacher conceptions about problem solving and problem-solving instruction. In G. Booker, P. Cobb, & T. de Mendicuti (Eds), *Proceedings of the 14th annual conference of the International Group for the Psychology of Mathematics Education* (pp. 135-142). Mexico: PME.

Groves, S. & Doig, B. (2002). Developing conceptual understanding: The role of the task in communities of mathematically inquiry. In A. D. Cockburn & E. Nardi (Eds), *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 25-32), University of East Anglia, Norwich.

Hembree, R. (1992). Experiments and relational studies in problem solving: a meta-analysis. *Journal for Research in Mathematics Education*, *23*(3), 242-73.

Lester, F. K. (1994). Musings about mathematical problem-solving research: 1970-1994. *Journal for Research in Mathematics Education*, *25*(6), 660-675.

Norton, S., McRobbie, C. J., & Cooper, T. J. (2002). Teachers' responses to an investigative mathematics syllabus: Their goals and practices. *Mathematics Education Research Journal*, *14*(1), 37-59.

New South Wales Department of Education (NSWDE) (1989*). Mathematics K-6*. Sydney: NSWDE.

Pegg, J. (1997). Mathematics teaching- Creating the future. In N. Scott & H. Hollingsworth (Eds), *Mathematics- Creating the future* (pp. 20-38). Melbourne: AAMT.

Peterson, P. L., Fennema, E., & Carpenter, T. P. (1987). *Teachers' pedagogical content beliefs in mathematics*. Paper presented at the AERA Conference, Washington, DC.

Schoenfeld, A. H. (1992). Learning to think mathematically: problem solving, metacognition and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan Publishing Co.

Stacey, K., & Groves, S. (1989). The teaching of applications, modelling and problem solving in Australia: 1984-1988. In W. Blum, M. Niss, & I. Huntley (Eds), *Modelling, applications and applied problem solving* (pp. 242-248). Chichester: Ellis Horwood.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap*. New York: The Free Press.

Thompson, A. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws, *Handbook of research on mathematics teaching and learning* (pp. 127-146), New York: Macmillan Publishing Company.